Superdeformed Bands of Odd Nuclei in A=190 Region in the Quasiparticle Picture

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Abstract

We study the properties of the superdeformed (SD) bands of ^{195}Pb and ^{193}Hg by the cranked Hartree-Fock-Bogoliubov method. Our calculations reproduce the flat behavior of the dynamical moment of inertia of two of the SD bands of ^{195}Pb measured recently. We discuss possible configuration assignments for the observed bands 3 and 4 of ^{195}Pb. We also calculate the two interacting SD bands of ^{193}Hg. Our analysis confirms the superiority of a density-dependent pairing force over a seniority pairing interaction.

The dynamical moment of inertia $\mathcal{J}$ of most superdeformed (SD) bands observed in nuclei of the $A \simeq 190$ region are increasing functions of the angular velocity ω[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. The recent discovery by Farris et al.,[12] that for the two lowest SD bands in ^{195}Pb, $\mathcal{J}$ is almost constant versus ω is therefore particularly interesting. In the same nucleus two other bands have been observed which display the usual increasing trend. It appears natural to attempt an explanation of these various behaviors of the SD bands of ^{195}Pb in terms of their quasiparticle (qp) structure. According to most theoretical investigations of the $A = 190$ region, the neutron qp's which are relevant for neutron numbers above the $N = 112$ gap are built on the [752]5/2, [512]5/2 and [624]9/2 orbitals[15, 16, 17, 18]. In this letter, we analyze the properties of the SD bands of the two odd-N neighbouring nuclei ^{195}Pb and ^{193}Hg [5] which today provide the richest information set on the neutron structure in the $A = 190$ superdeformed well. Our work is based on the cranked Hartree-Fock-Bogoliubov (HFB) approach which has been shown to reproduce with good accuracy the SD band properties

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of even nuclei[19, 20]. As in ref. [18], the mean-field method has been corrected by means of the Lipkin-Nogami prescription[21, 22, 23] to take into account the finite number of nucleons. The nucleon-nucleon effective interaction in the particle-hole channel is the Skyrme force within the Skm* parametrization[24]. In the pairing channel we use a zero-range force with a surface-peaked density dependence as described in ref.[20]. In a previous study of the yrast SD band of ^{194}Pb, this type of force was shown to improve significantly the alignment properties which determine the saturation of $\mathcal{J}$ for large values of ω.

Our method of solution of the HFB equation combines the imaginary-time evolution method to determine the basis which diagonalizes the mean-field hamiltonian and a diagonalization of the HFB hamiltonian matrix to construct the canonical basis. Details are available in ref.[18]. The different bands of the odd nuclei we are concerned with, are described by the self-consistent creation of the appropriate qp [25] on the even vacuum. This requires some care in the numerical treatment, since, except at zero angular velocity, parity and signature are the only quantum numbers available for sorting qp's. Therefore, we rely mostly on continuity properties versus ω to follow a given SD band.

The first part of this letter is devoted to a study of $\mathcal{J}$ and qp routhian (qpr) properties of ^{195}Pb. In the second part we analyze how the structure of the pairing force affects SD properties by comparing the results for ^{193}Hg with the density dependent interaction with those of ref.[25] in which a seniority interaction was used.

In Fig. 1 we compare the dynamical moments of inertia $\mathcal{J}$ of the four observed bands[12] with those calculated for the seven SD bands built on the [752]5/2,α=±1/2, [512]5/2,α=±1/2, [624]9/2,α=±1/2 and [642]3/2, α=−1/2 orbitals. The SD bands built on the intruder ν[752]5/2 bands display a small variation of $\mathcal{J}$ for $\hbar\omega \geq 0.24$ MeV. It seems therefore reasonable to assign them to the first and second experimental bands. The observed significant signature splitting is also reproduced and suggest that the band with lowest moment of inertia has a positive signature. For bands 3 and 4 the authors of ref.[12] have argued that they may be built on ν[624]9/2 orbitals. On the other hand, the theoretical part of their analysis indicated that transition energies associated with ν[624]9/2 and ν[512]5/2 bands would be almost identical. This is supported by our results. Indeed we find that the moment of inertia of the ν[624]9/2 bands is very similar to that of the ν[512]5/2 bands for $\hbar\omega \geq 0.24$ MeV. Both sets of $\mathcal{J}$ agree qualitatively with those observed for bands 3 and 4. Based on the sole information given by the moment of inertia, it is therefore not possible to decide whether bands 3 and 4 should be labeled ν[624]9/2 or ν[512]5/2. We only note a small signature splitting for the ν[512]5/2 when $\hbar\omega \geq$ 0.3 MeV A complementary piece of information is provided by the excitation energy $E_{\rm rel}$ of the bands with respect to each other. It is generally believed that this quantity which is directly available in the calculation (see Table 1) is correlated with the observed relative population of the bands. In Table 1, the reference energy corresponds to the state of ν[752]5/2, α=−1/2 band of the angular momentum $I = 32.5$, which was calculated by averaging two energies of $I = 31.5$ and 33.5. $E_{\rm rel}$ of the other negative-signature bands were calculated in the same way. According to Table 1, the ν[752]5/2 bands are the lowest, the ν[512]5/2 bands are second lowest and the ν[624]9/2 bands are the most excited. Therefore, for the ν[752]5/2 bands, the agreement of our calculation with experiment concerns both the magnitude and behavior of moments of inertia and the excitation energy. On the other hand, we would be led to assign a ν[512]5/2 structure to bands 3 and 4. We note however that the energy differences in Table 1 are of the order of 0.1MeV. Such a precision could well be below the limit of physical credibility

band	$E_{\rm rel}$ [MeV]
ν[752]5/2, α=+1/2	0.125
ν[752]5/2, α=−1/2	0.000
ν[512]5/2, α=+1/2	0.168
ν[512]5/2, α=−1/2	0.172
ν[624]9/2, α=+1/2	0.297
ν[624]9/2, α=−1/2	0.297
ν[642]3/2, α=−1/2	0.368

Table 1: Calculated relative excitation energies $E_{\rm rel}$ of the six SD bands in ^{195}Pb at $I = 32.5$. The reference band is ν[752]5/2, α=−1/2 band. $E_{\rm rel}$ of the negative-signature bands were calculated from averages of energies of $I = 31.5$ and 33.5.

of a calculation such as ours when it comes to the relative position of orbitals. We shall return to this point when discussing the crossing phenomenon in ^{193}Hg.

The charge quadrupole moments $Q_{\rm c}$ of the ν[512]5/2 and ν[624]9/2 bands are shown in Fig. 2. They differ by approximately 0.3b which is probably too small to be measured. As expected for bands with large m values no signature splitting is found. The quadrupole moments of the intruder bands are more different: 19.96 eb at $I = 32.5$ ($\hbar\omega = 0.314$MeV) for $\alpha = +1/2$ and 19.47 eb at $I = 31.5$ ($\hbar\omega = 0.290$MeV) for its signature partner. Differences between the magnetic moments of the ν[512]5/2 and ν[624]9/2 bands would lead to different crosstalks between the bands. However their values are rather similar ($\simeq 12.4\ e\hbar/2M_{\rm p}c$ at $I \simeq 31$, $M_{\rm p}$ being the proton mass) for the four bands. They therefore also do not provide a convenient signature to establish the nature of the 3rd and 4th SD bands.

Let us now consider the evolution versus ω of the qpr's. Because the mean-fields are self-consistently modified by the creation of a qp, they are not the same for the six SD bands and differ also from those calculated for ^{194}Pb. In Fig. 3 and 4 we show the neutron qpr's for the ν[752]5/2, $\alpha = -1/2$ (after a crossing with [512]5/2,α=−1/2) and ν[624]9/2, $\alpha = +1/2$ bands respectively. In each figure, the thick curve indicates which qp is occupied. One sees that the difference between the occupied and empty qpr's of the signature partners is large for all values of the rotational frequency. This is caused by the modification of the time-odd components of the mean-field that is generated by the occupation of only one of the signature partner ; in particular one notes that the difference does not vanish at $\omega = 0$. Figs. 3 and 4 show that the flat $\mathcal{J}$ behavior for the ν[752]5/2 bands is strongly correlated with the curvature of the associated routhians which is markedly different from those of other qpr's. Moreover, the fact that the average curvature of the ν[752]5/2, $\alpha = +1/2$ routhian is the smallest, is consistent with the low moment of inertia of the corresponding band.

The accident in the theoretical $\mathcal{J}$ for the [752]5/2 and [512]5/2 bands for 0.1MeV$\leq \hbar\omega \leq$0.2MeV is generated by a band crossing. Such an accident can always happen when two crossing bands have the same quantum numbers. In case of a band crossing, our convention is to denote bands with the qp configuration which characterizes them for large values of $\hbar\omega$. For values of $\hbar\omega$ near 0.15 MeV, both for the ν[752]5/2 and ν[512]5/2 bands, we have not been able to obtain solutions satisfying the angular-momentum constraint accurately. Indeed, we have met numerical instabilities caused by the near degeneracy both in energy and angular momentum. A correct physical solution of this problem requires a self-consistent configuration mixing calculation which is beyond the scope of this study. As a band crossing has not been observed in

^{195}Pb, we infer that the calculation shown in Fig. 3 overestimates the energy difference between the $\nu[752]5/2$, α=$-1/2$ and $\nu[512]5/2$, α=$-1/2$ energies at $\omega = 0$ by at least 0.1MeV. One way to remedy this deficiency, would be to correct the mean-field in such a way that the energy of the $\nu[512]5/2$, α=$-1/2$ is pushed up, leading to a crossing below the lowest observed $\hbar\omega$. According to the above discussion, the associated excitation energy of the $\nu[512]5/2$ SD bands would increase and possibly modify our assignation for band 3 and 4, leading us to agree with the conclusions of ref.[12].

We should also mention the relative position of $\nu[624]9/2$, α=$+1/2$ in Fig. 4. The qpr is not the lowest in routhians having the positive parity and positive signature. It is anticipated, however, that a particle-type qpr ($\nu[624]9/2$) becomes lower than a hole-type one ($\nu[642]3/2$) in ^{195}Pb, when their energies are comparable for the yrast SD band of ^{194}Pb. Given the present uncertainty of mean-field calculation concerning the detailed relative location of the qpr's, we cannot disregard $\nu[624]9/2$ as one of candidates of the configuration of bands 3 and 4.

On Fig. 2 and in Table 1 we have also reported result for the negative signature band built on the $\nu[642]3/2$,α=$-1/2$ quasiparticle. Although the routhian of this state is lower than that of the $\nu[624]9/2$ in the quasiparticle spectrum of ^{194}Pb, Table 1 shows that the [642]3/2 SD band is more excited. This is a consequence of self-consistency effects.

Let us now see how these considerations can be extended to the analysis of the nucleus ^{193}Hg. So far 6 SD bands including two identical bands have been observed [5]. Within the HFB method we have already performed a study using the same Skyrme force parametrization for the mean-field[25]. On the other hand, in ref. [25] pairing correlations have been described with a seniority interaction. The results of analysis limited to the two interacting bands $\nu[752]5/2$, $\alpha = -1/2$ band (band 4) and $\nu[512]5/2$, $\alpha = -1/2$ band (band 1) with a zero-range density dependent pairing force are shown in Fig. 5 together with the experimental data. For each band a separate HFB calculation has been performed. Our calculation shows that this dual self-consistent HFB analysis is able to reproduce the observed band interaction. This is a significant improvement over the calculation of ref. [25] in which no interaction was found, and it provides an additional indication of the superiority of a surface-type zero-range pairing force over a seniority interaction. On the other hand, the angular velocity at which bands interact is found at $\hbar\omega \simeq 0.15$MeV instead of the observed value $\hbar\omega \simeq 0.25$MeV. This difference may reflect an inaccuracy of the relative location of the relevant qpr's in ^{193}Hg which would then be consistent with our discussion on the position of orbitals [752]5/2 and [512]5/2 in ^{195}Pb.

In summary, we have analyzed the properties of the SD bands of two odd-N nuclei ^{195}Pb and ^{193}Hg by making use of the cranked HFB method. Our self-consistent calculation has confirmed the general belief that the flat behavior of $\mathcal{J}$ in bands 1 and 2 of ^{195}Pb is related to the curvature of the neutron intruder qpr. For bands 3 and 4 we have found that configurations based on the [624]9/2 and [512]5/2 are in competition. Both the moment of inertia, the quadrupole moments and the magnetic moments of these four SD bands are very similar. In particular, within the HFB method these quantities do not provide an effective mean of deciding the nature of the observed bands 3 and 4. We have also calculated with qualitative success the two interacting SD bands of ^{193}Hg. The results of this analysis provide additional support for an effective pairing force acting predominantly at the nuclear surface. The quantitative inaccuracy on the position of the crossing frequencies ($\Delta\hbar\omega \approx 0.1$ MeV) could be an indication that qpr associated with the [512]5/2 is too low by 0.1 MeV relative to the rest of the spectrum. It is an interesting question whether it is possible to determine an effective interaction with the same

global qualities of the SkM* force which could also achieve a better precision as regards the single-particle energies.

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Figure Captions

Fig. 1 a) Experimental dynamical moments of inertia of the four SD bands [12] of ^{195}Pb.

b) HFB dynamical moments of inertia of the lowest SD bands of ^{195}Pb. (the negative signature bands are denoted with black symbols)

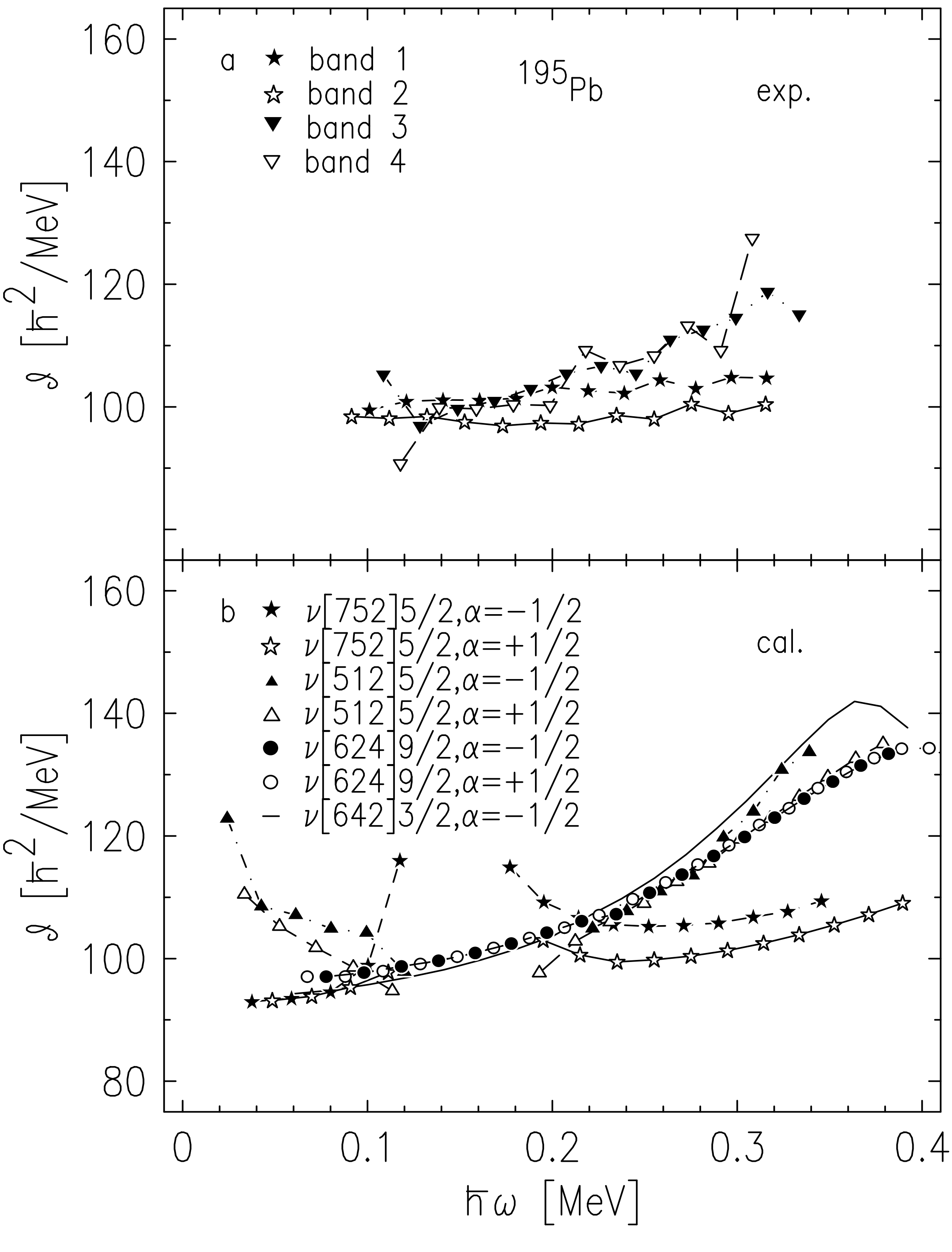
Fig. 2 Calculated charge quadrupole moments $Q_{\rm c}$ of the ν[512]5/2, ν[624]9/2 bands and the ν[642]3/2,$\alpha = -1/2$ band of ^{195}Pb. Correspondence between the symbols and the SD bands is the same as in Fig. 1b.

Fig. 3 Neutron qpr of ν[752]5/2, α=$-$1/2 SD band. The thick dotted curve indicates the occupied qpr. Full (dashed) curves correspond to positive parity and positive (negative) signature, and dot-dashed (dotted) curves to negative parity and positive (negative) signature.

Fig. 4 Neutron qpr of ν[624]9/2, α=+1/2 SD band. The thick full curve corresponds to the occupied qpr. The other drawing conventions are the same as those used in Fig. 3.

Fig. 5 Experimental dynamical moment of inertia $\mathcal{J}$ of ^{193}Hg for bands 1 (solid triangle) and 4 (solid circle). Our results are indicated by open triangles for the ν[512]5/2, α=$-$1/2 configuration and by open circles for the ν[752]5/2, α=$-$1/2 one.

Fig.1,


a ★ band 1
☆ band 2
▼ band 3
▽ band 4
195Pb
exp.
b ★ ν[752]5/2,α=−1/2
☆ ν[752]5/2,α=+1/2
▲ ν[512]5/2,α=−1/2
△ ν[512]5/2,α=+1/2
● ν[624]9/2,α=−1/2
○ ν[624]9/2,α=+1/2
— ν[642]3/2,α=−1/2
cal.
ℑ [ħ²/MeV]
ħω [MeV]
160
140
120
100
80
0
0.1
0.2
0.3
0.4

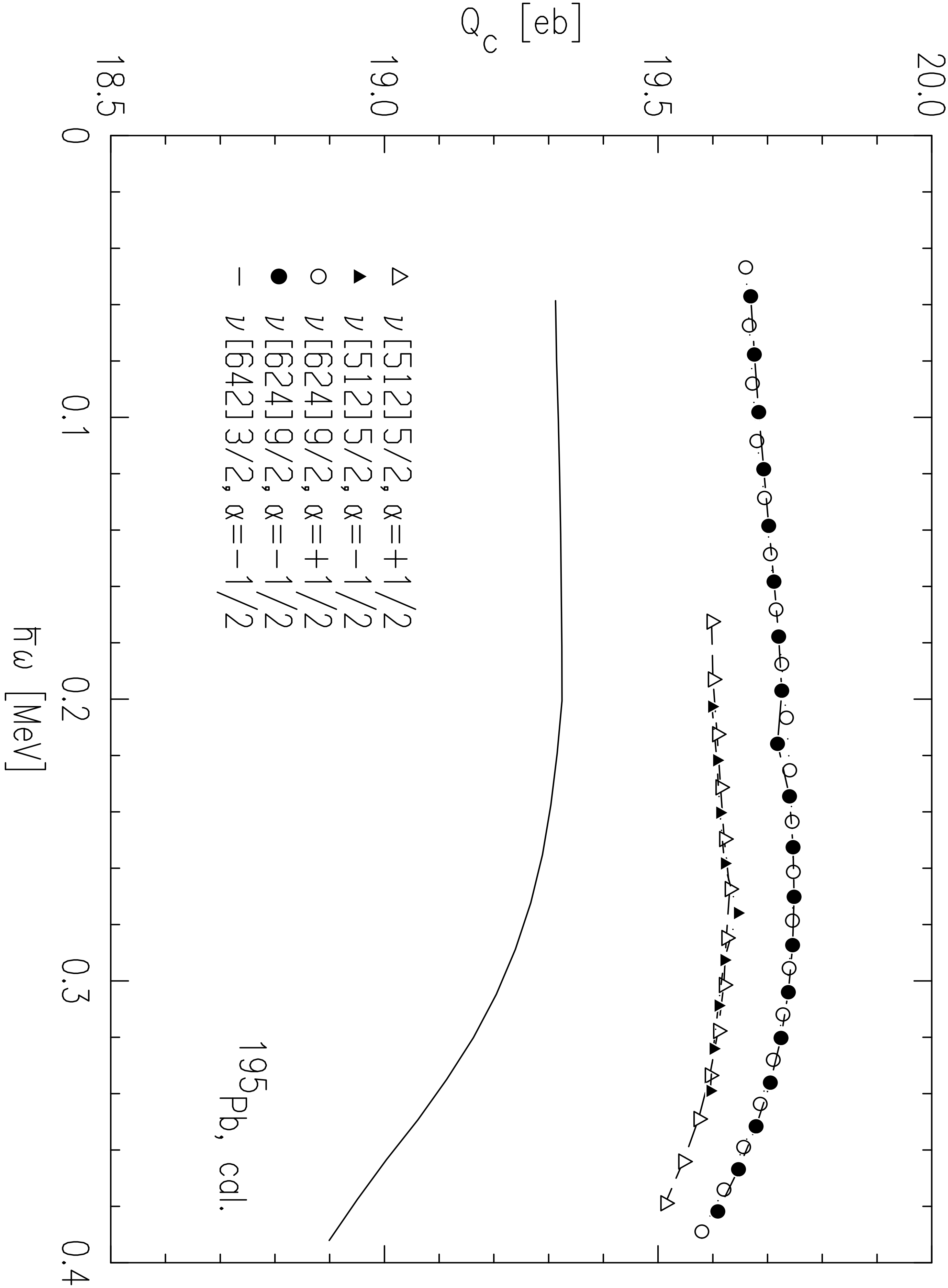
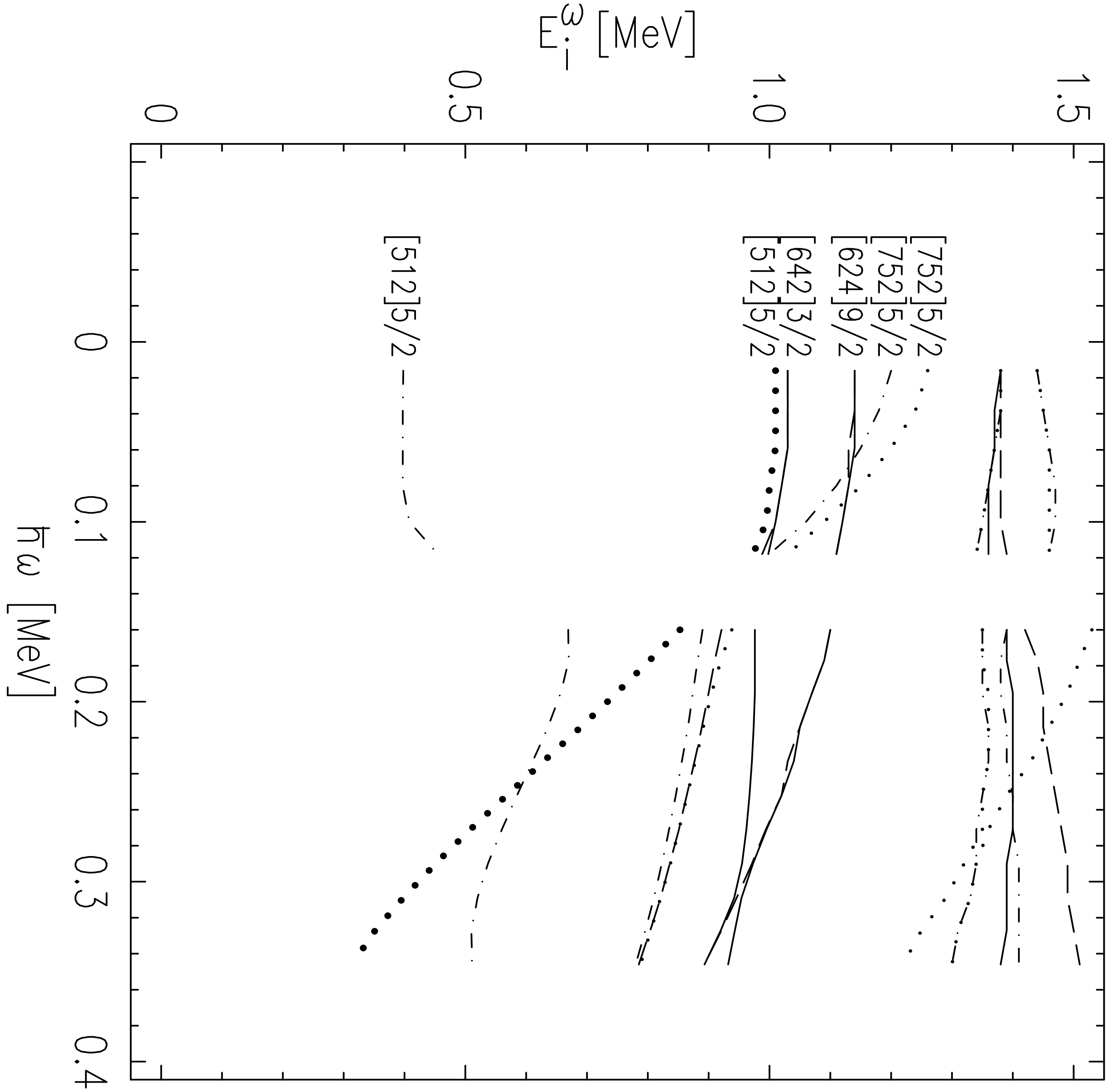

Q_c [eb]
18.5
19.0
19.5
20.0
0
0.1
0.2
0.3
0.4
ħω [MeV]
ν[512]5/2, α=+1/2
ν[512]5/2, α=−1/2
ν[624]9/2, α=+1/2
ν[624]9/2, α=−1/2
ν[642]3/2, α=−1/2
195Pb, cal.

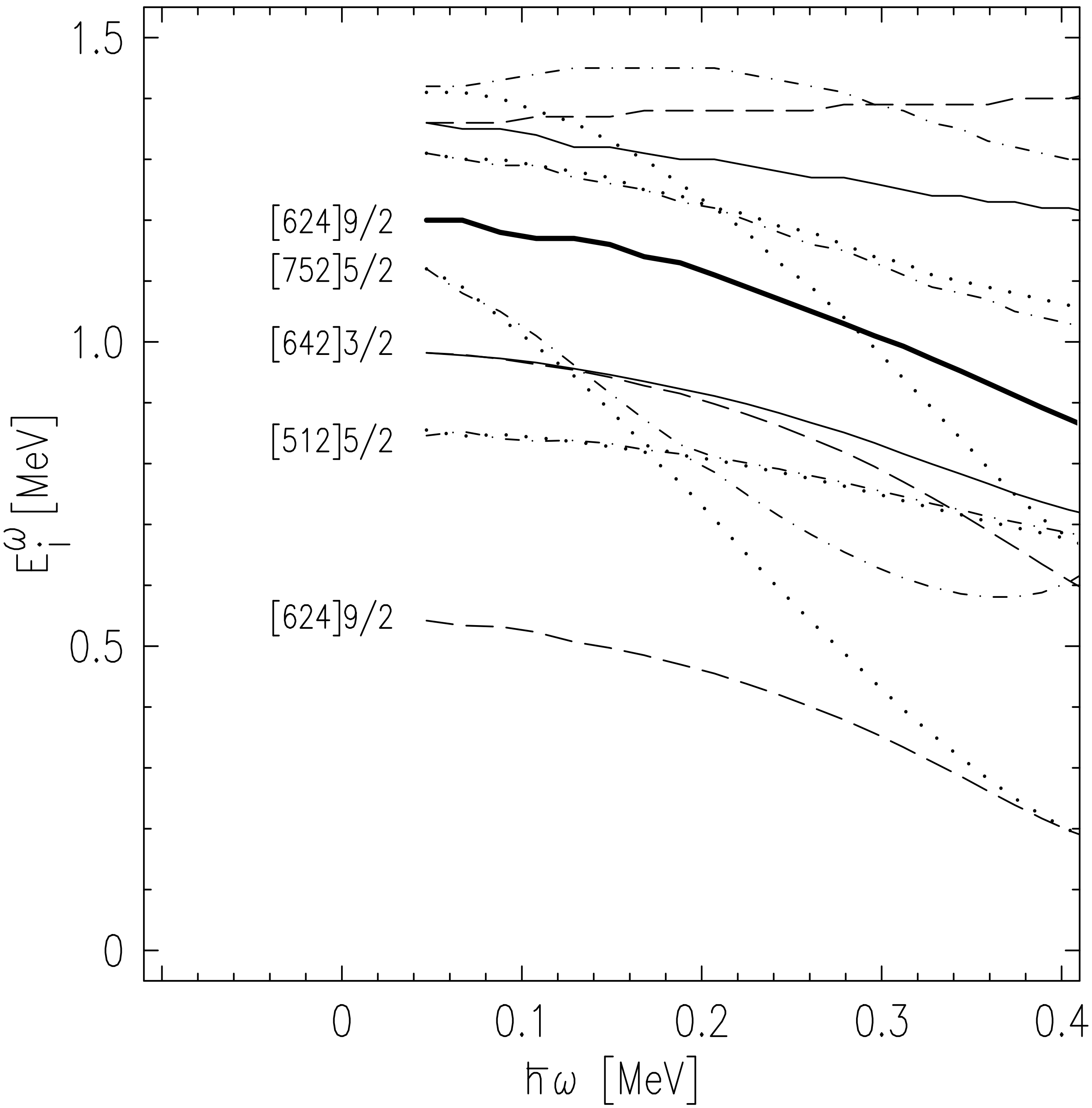
Fig.2

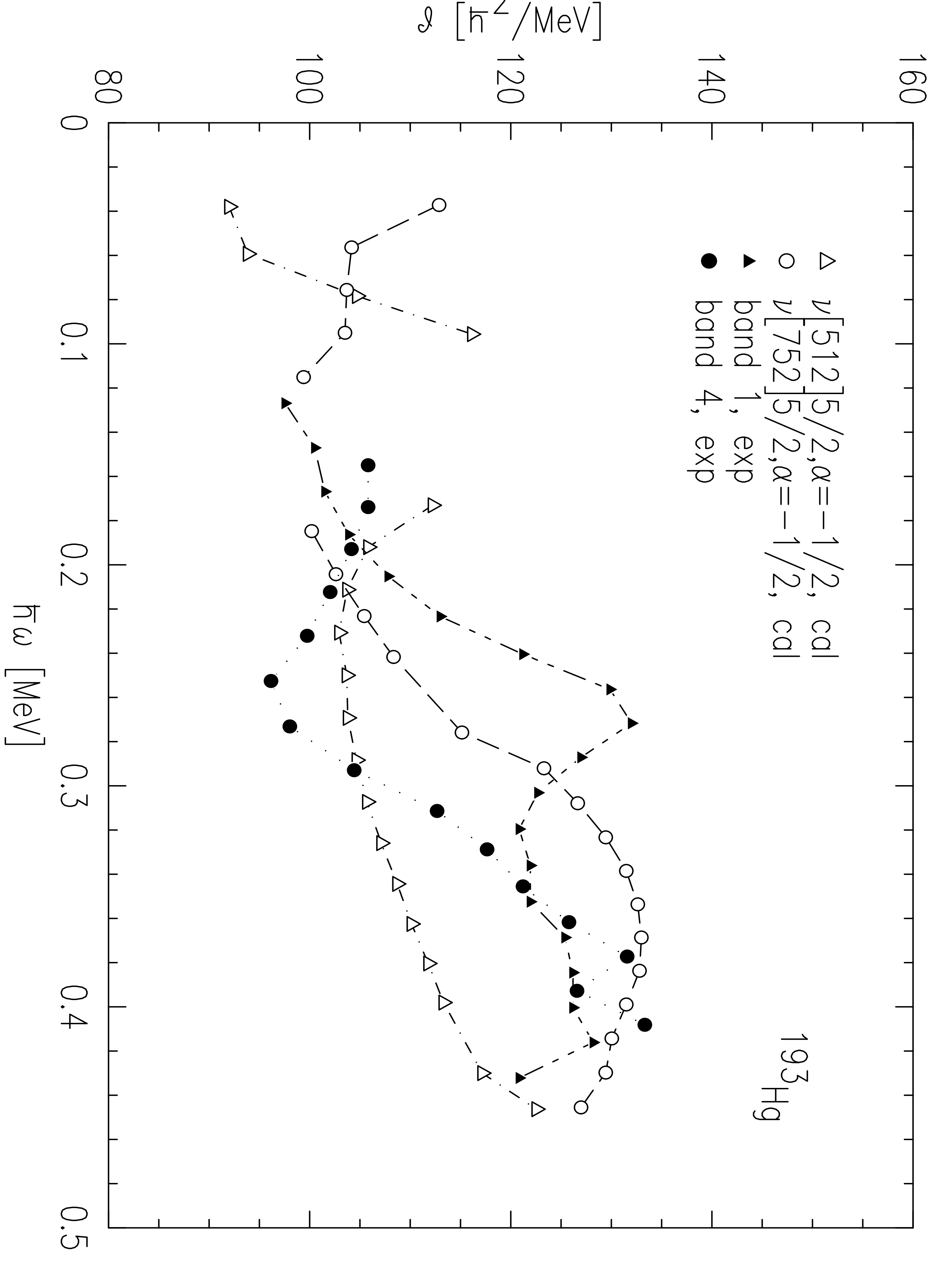


E_i^{ω} [MeV]
$\hbar\omega$ [MeV]
0
0.5
1.0
1.5
0
0.1
0.2
0.3
0.4
[752]5/2
[752]5/2
[624]9/2
[642]3/2
[512]5/2
[512]5/2

Fig.3

Fig.4


[624]9/2
[752]5/2
[642]3/2
[512]5/2
[624]9/2
1.5
1.0
0.5
0
E_i^{ω} [MeV]
0
0.1
0.2
0.3
0.4
$\hbar\omega$ [MeV]



ℐ [ħ^2/MeV]
80
100
120
140
160
0
0.1
0.2
0.3
0.4
0.5
ħω [MeV]
ν[512]5/2,α=−1/2, cal
ν[752]5/2,α=−1/2, cal
band 1, exp
band 4, exp
^{193}Hg

Fig.5